MSc Research Presentation

Constraining the mass of a fermionic dark matter particle via phase space bounds in dwarf spheroidal galaxies

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Introduction

- Evidence for non-luminous matter from galactic rotation curves, large scale structure of the universe, gravitational lensing, etc
- Local Group (LG) dwarf spheroidal galaxies (dSphs) = dark matter dominated satellites to the Milky Way or M31

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Structure

- Tremaine-Gunn Bound

  A model robust lower mass bound for known dark matter density profile

- Density Profile Models
  - Ruffini-Argüelles-Rueda (RAR) model

    with inconsistencies
  - Jeans analysis with Virial parameters
  - Read-Steger profiles

    based on Jeans+Virial analysis

- Mass bound with Read-Steger profiles

The Tremaine-Gunn Bound I

- Limiting condition for the phase space density f(q,p), derived from the fully degenerate Fermi gas[1]

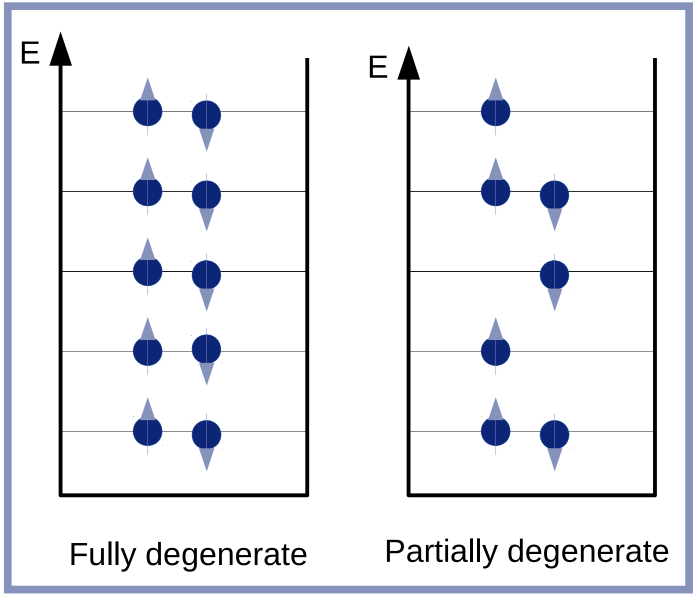
(1.1) $$f(q,p) \leqslant \frac{g}{(2\pi\hbar)^3}$$

- The phase space density is difficult to access in practice, but coarse-grained phase space densities $\tilde{f}$(r,p) obey the same limit[2]

(1.2) $$\hat{f}(q,p) = \frac{1}{vol(\Delta\Pi)} \int_{\Delta\Pi(p,q)} d\Pi' f(p',q')$$

- $\tilde{f}$(r) as obtained by integrating f(r,p) over momentum space reads

(1.3) $$\hat{f}(r) = \frac{\rho(r)}{m V(r)_{ms}} = \frac{\rho(r)}{m^4 \, 4/3 \pi v_{max}^3(r)}$$



The Tremaine-Gunn Bound II

- Together, equations (1.1) and (1.3) yield the condition for the dark matter particle mass

(1.4)
$$\boxed{m \geq \left(\frac{\hbar^3 6\pi^2 \rho(r)}{g\, v_{max}^3(r)} \right)^{1/4}}$$

- Using the escape velocity as an estimate for $v_{max}(r)$ we only need to know the density profile $\rho(r)$.
- Previous estimation for LG dSphs[2]

$$\boxed{m \geq 269\, eV}$$

Density profiles – The RAR model I [3][4]

- The Ruffini-Argüelles-Rueda (RAR) model is a dark matter model modified by 3 free parameters.

(2.1) $$T^{\mu\nu}=(P+\rho)u^{\mu}u^{\nu}-P\,g^{\mu\nu}$$

(2.2) $$ds^2=e^{\nu(r)}dt^2-e^{\mu(r)}(dr^2+r^2 d\Omega^2)$$

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(2.4) $$dU = T\,dS - P\,dV + \mu\,dN$$

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(2.7) $$0 = \left(\frac{\rho + P - \mu\,\eta}{T}\right)\partial_i T - \partial_i P + \eta\partial_i \mu$$

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$$\boxed{(2.8)\quad \left(\frac{\partial_i T}{T}+\frac{\partial_i \nu}{2}\right)(\rho+P)+\left(\frac{\partial_i \mu}{\mu}-\frac{\partial_i T}{T}\right)\eta\mu=0}$$

$$\left\{\begin{aligned} &(2.6)\quad \partial_i P+\frac{P+\rho}{2}\partial_i \nu(r)=0 \\ &(2.7)\quad 0=\left(\frac{\rho+P-\mu\eta}{T}\right)\partial_i T-\partial_i P+\eta\partial_i\mu \end{aligned}\right.$$

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$$\text{(2.8)} \quad \boxed{\left(\frac{\partial_i T}{T} + \frac{\partial_i \nu}{2}\right)(\rho+P) + \left(\frac{\partial_i \mu}{\mu} - \frac{\partial_i T}{T}\right)\eta\mu = 0}$$

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$$\text{(2.10)} \quad \mu = \mu_0 e^{\frac{\nu_0-\nu}{2}}$$

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- Fermi statistics relate density ρ(r) and pressure P(r) to temperature T(r) and chemical potential μ(r)

(2.11) $$f(p,r)=\left(1+\exp\left(\frac{E(p)}{kT(r)}-\frac{\mu(r)}{kT(r)}\right)\right)^{-1}$$

(2.12) $$\rho(r)=2\frac{1}{h^3}\int f(p,r)E(p)d^3p$$

(2.13) $$P(r)=2\frac{1}{3h^3}\int f(p,r)\frac{p^2}{E(p)}d^3p$$

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- Tolman–Oppenheimer–Volkoff equation

(2.14) $$\frac{d\nu}{dr} = 2\left[2GM(r) + 4\pi G r^3 P(r)\right]\left[r^2\left(1 - 2GM(r)/r\right)\right]^{-1}$$

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- Enclosed mass

(2.15) $$\frac{dM}{dr}=4\pi r^2\rho$$

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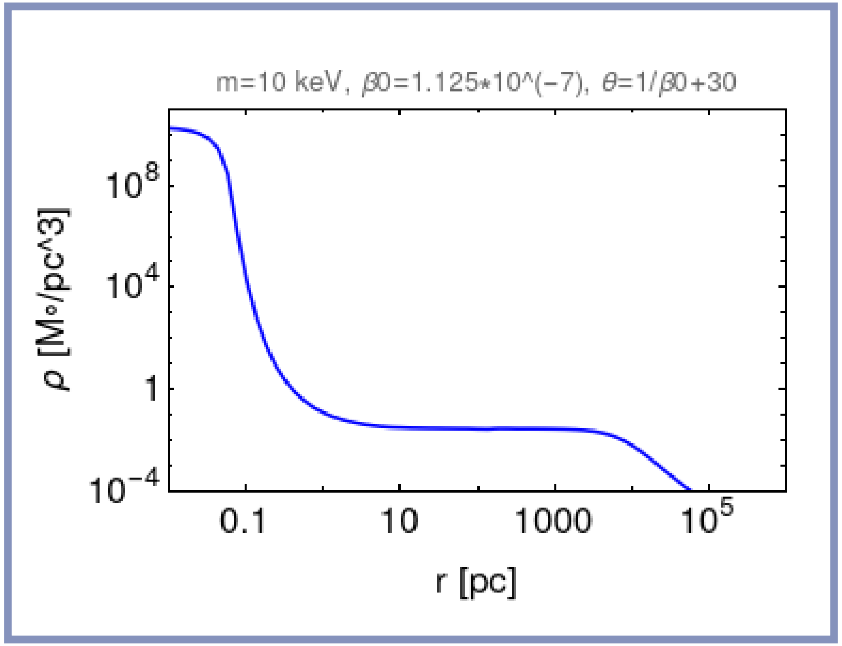

- Issues in the non-relativistic limit

$$\rho_e(r)=\rho_m(r)=\rho(r)=m\,\eta(r)$$

$$2\frac{1}{h^3}\int f(p,r)E(p)d^3p\approx 2\frac{m}{h^3}\int f(p,r)d^3p$$

$$\sqrt{m^2+p^2}\approx m \qquad p^2\ll 1$$

Density profiles – The RAR model III

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$$P(r)=2\frac{1}{3h^3}\int f(p,r)\frac{p^2}{E(p)}d^3p$$

$$P(r)\ll\rho(r)$$

Density profiles – The RAR model III

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$$\left(\frac{\partial_i T}{T}+\frac{\partial_i\nu}{2}\right)(\rho+P)+\left(\frac{\partial_i\mu}{\mu}-\frac{\partial_i T}{T}\right)\eta\mu=0$$

(2.8) This means that profiles are linearly dependent

$$\left(m\frac{\partial_i T}{T}+m\frac{\partial_i\nu}{2}+\partial_i\mu-\frac{\partial_i T}{T}\mu\right)\eta=0$$

Profiles are special cases, resulting from (arbitrary) assumptions

- Issues in the general case

(2.11) $$f(p,r)=\left(1+\exp\left(\frac{E(p)}{kT_0}\exp\left(\frac{\nu(r)-\nu_0}{2}\right)-\frac{\mu_0}{kT_0}\right)\right)^{-1}$$

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Density profiles – The RAR model IV

- Issues in the general case

(2.11) $$f(p,r)=\left(1+\exp\left(\frac{E(p)}{kT_0}\exp\left(\frac{\nu(r)-\nu_0}{2}\right)-\frac{\mu_0}{kT_0}\right)\right)^{-1}$$

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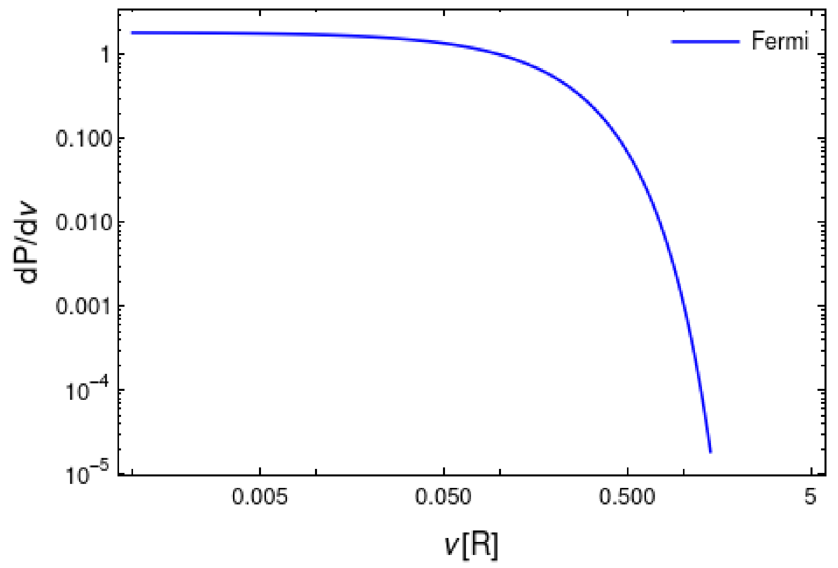

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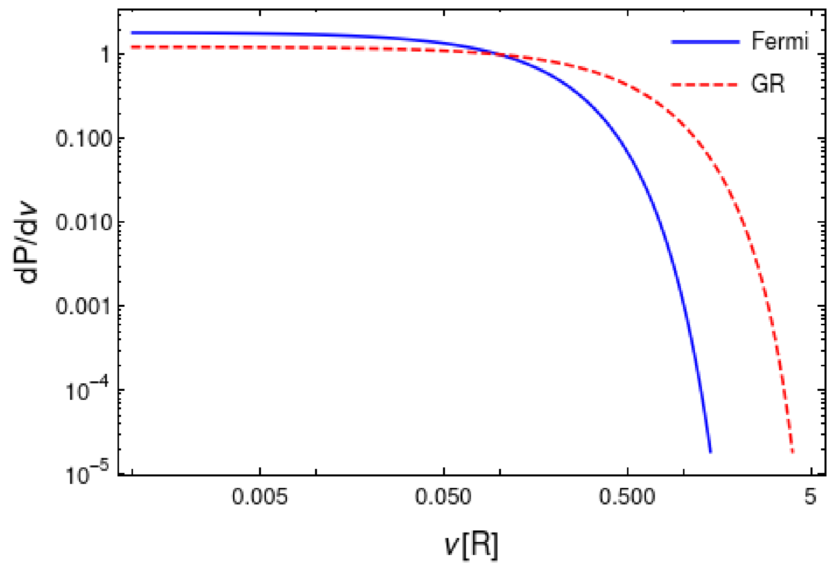
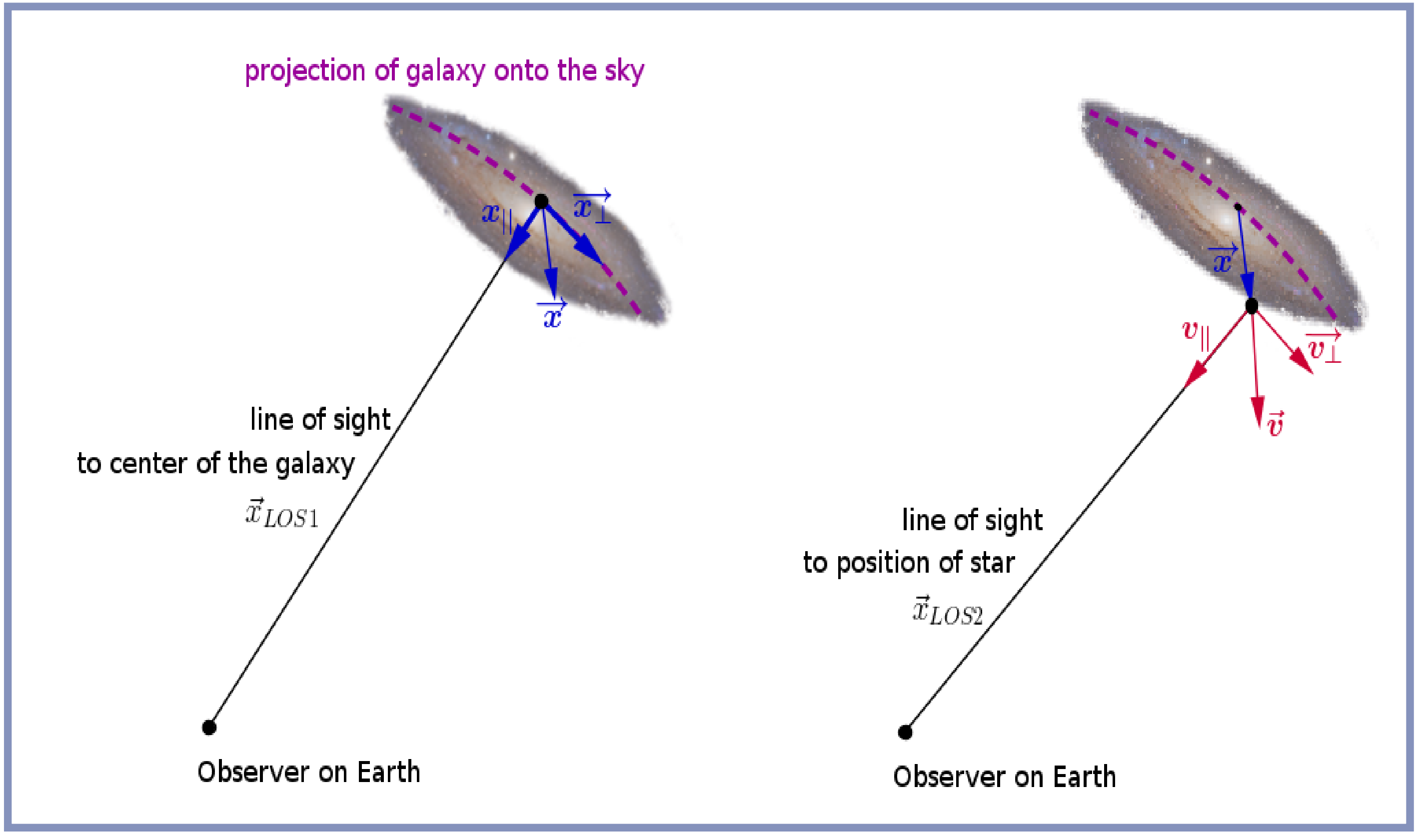


Profiles violate energy-momentum conservation



$$\int_{galaxy} f(q,p)\, d^3q\, d^3p = 1$$

Density profiles – Jeans analysis II [5]

- Boltzmann equation in spherical coordinates

$$\frac{\partial f}{\partial t}+p_r\frac{\partial f}{\partial r}+\frac{p_\theta}{r^2}\frac{\partial f}{\partial \theta}+\frac{p_\phi}{r^2\sin^2\theta}\frac{\partial f}{\partial \phi}-\left(\frac{\partial \Phi}{\partial r}-\frac{p_\theta^2}{r^3}-\frac{p_\phi^2}{r^3\sin^2\theta}\right)\frac{\partial f}{\partial p_r}-\left(\frac{\partial \Phi}{\partial \theta}-\frac{p_\phi^2\cos\theta}{r^2\sin^3\theta}\right)\frac{\partial f}{\partial p_\theta}-\frac{\partial \Phi}{\partial \phi}\frac{\partial f}{\partial p_\phi}=0 \tag{3.1}$$

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Integrals over momentum coordinates

Jeans equations

$$(3.2)\qquad \frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial\Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

$$(3.3)\qquad \frac{\partial}{\partial r}(\rho\overline{v_r^4})+\frac{2\beta'}{r}\rho\overline{v_r^4}+3\rho\frac{d\Phi}{dr}\overline{v_r^2}=0$$

$$(3.4)\qquad 4\rho\overline{v_r^2v_t^2}+r\frac{\partial}{\partial r}(\rho\overline{v_r^2v_t^2})+\rho\frac{\partial\Phi}{\partial r}\overline{v_t^2}r-\rho\overline{v_t^4}=0$$

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profiles

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Integrals over momentum coordinates

Jeans equations

$$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial \Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0 \tag{3.2}$$

$$\frac{\partial}{\partial r}(\rho\overline{v_r^4})+\frac{2\beta'}{r}\rho\overline{v_r^4}+3\rho\frac{d\Phi}{dr}\overline{v_r^2}=0 \tag{3.3}$$

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profiles

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profiles

Integrals over spatial+momentum coordinates

Virial equations

(3.5) $$3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}R\,dR=2\int_0^\infty \rho(r)\frac{d\Phi(r)}{dr}r^3dr$$

(3.6) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3dr$$

(3.7) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R^3dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5dr$$

parameters

Density profiles – Jeans analysis II[5]

- Boltzmann equation in spherical coordinates

(3.1) $$\frac{\partial f}{\partial t}+p_r\frac{\partial f}{\partial r}+\frac{p_\theta}{r^2}\frac{\partial f}{\partial \theta}+\frac{p_\phi}{r^2\sin^2\theta}\frac{\partial f}{\partial \phi}-\left(\frac{\partial \Phi}{\partial r}-\frac{p_\theta^2}{r^3}-\frac{p_\phi^2}{r^3\sin^2\theta}\right)\frac{\partial f}{\partial p_r}-\left(\frac{\partial \Phi}{\partial \theta}-\frac{p_\phi^2\cos\theta}{r^2\sin^3\theta}\right)\frac{\partial f}{\partial p_\theta}-\frac{\partial \Phi}{\partial \phi}\frac{\partial f}{\partial p_\phi}=0$$

Integrals over momentum coordinates

Jeans equations

(3.2) $$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial \Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}\,2\beta=0$$

(3.3) $$\frac{\partial}{\partial r}(\rho\overline{v_r^4})+\frac{2\beta'}{r}\rho\overline{v_r^4}+3\rho\frac{d\Phi}{dr}\overline{v_r^2}=0$$

(3.4) $$4\rho\overline{v_r^2 v_t^2}+r\frac{\partial}{\partial r}(\rho\overline{v_r^2 v_t^2})+\rho\frac{\partial \Phi}{\partial r}\overline{v_t^2}\,r-\rho\overline{v_t^4}=0$$

$$\beta=-(\overline{v_\theta^2}+\overline{v_\phi^2})/(2\overline{v_r^2})+1 \qquad \beta'=1-3\overline{v_r^2 v_t^2}/(2\overline{v_r^4})$$

profiles

Integrals over spatial+momentum coordinates

Virial equations

(3.5) $$3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}\,R\,dR=2\int_0^\infty \rho(r)\frac{d\Phi(r)}{dr}r^3\,dr$$

(3.6) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}\,R\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3\,dr$$

(3.7) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}\,R^3\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5\,dr$$

parameters

Density profiles – Jeans analysis II [5]

- Boltzmann equation in spherical coordinates

(3.1) $$\frac{\partial f}{\partial t}+p_r\frac{\partial f}{\partial r}+\frac{p_\theta}{r^2}\frac{\partial f}{\partial \theta}+\frac{p_\phi}{r^2\sin^2\theta}\frac{\partial f}{\partial \phi}-\left(\frac{\partial \Phi}{\partial r}-\frac{p_\theta^2}{r^3}-\frac{p_\phi^2}{r^3\sin^2\theta}\right)\frac{\partial f}{\partial p_r}-\left(\frac{\partial \Phi}{\partial \theta}-\frac{p_\phi^2\cos\theta}{r^2\sin^3\theta}\right)\frac{\partial f}{\partial p_\theta}-\frac{\partial \Phi}{\partial \phi}\frac{\partial f}{\partial p_\phi}=0$$

Integrals over momentum coordinates

Jeans equations

(3.2) $$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial \Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}\,2\beta=0$$

(3.3) $$\frac{\partial}{\partial r}(\rho\overline{v_r^4})+\frac{2\beta'}{r}\rho\overline{v_r^4}+3\rho\frac{d\Phi}{dr}\overline{v_r^2}=0$$

(3.4) $$4\rho\overline{v_r^2v_t^2}+r\frac{\partial}{\partial r}(\rho\overline{v_r^2v_t^2})+\rho\frac{\partial \Phi}{\partial r}\overline{v_t^2}\,r-\rho\overline{v_t^4}=0$$

$$\beta=-(\overline{v_\theta^2}+\overline{v_\phi^2})/(2\overline{v_r^2})+1 \qquad \beta'=1-3\overline{v_r^2v_t^2}/(2\overline{v_r^4})$$

profiles

Integrals over spatial+momentum coordinates

Virial equations

(3.5) $$3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}\,R\,dR=2\int_0^\infty \rho(r)\frac{d\Phi(r)}{dr}r^3\,dr$$

(3.6) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}\,R\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3\,dr$$

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parameters

- Boltzmann equation in spherical coordinates

(3.1)
$$\frac{\partial f}{\partial t}+p_r\frac{\partial f}{\partial r}+\frac{p_\theta}{r^2}\frac{\partial f}{\partial\theta}+\frac{p_\phi}{r^2\sin^2\theta}\frac{\partial f}{\partial\phi}-\left(\frac{\partial\Phi}{\partial r}-\frac{p_\theta^2}{r^3}-\frac{p_\phi^2}{r^3\sin^2\theta}\right)\frac{\partial f}{\partial p_r}-\left(\frac{\partial\Phi}{\partial\theta}-\frac{p_\phi^2\cos\theta}{r^2\sin^3\theta}\right)\frac{\partial f}{\partial p_\theta}-\frac{\partial\Phi}{\partial\phi}\frac{\partial f}{\partial p_\phi}=0$$

Integrals over momentum coordinates

Integrals over spatial+momentum coordinates

Jeans equations

(3.2)
$$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial\Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

(3.3)
$$\frac{\partial}{\partial r}(\rho\overline{v_r^4})+\frac{2\beta'}{r}\rho\overline{v_r^4}+3\rho\frac{d\Phi}{dr}\overline{v_r^2}=0$$

(3.4)
$$4\rho\overline{v_r^2v_t^2}+r\frac{\partial}{\partial r}(\rho\overline{v_r^2v_t^2})+\rho\frac{\partial\Phi}{\partial r}\overline{v_t^2}r-\rho\overline{v_t^4}=0$$

$$\beta=-(\overline{v_\theta^2}+\overline{v_\phi^2})/(2\overline{v_r^2})+1 \qquad \beta'=1-3\overline{v_r^2v_t^2}/(2\overline{v_r^4})$$

Virial equations

(3.5)
$$3\int_0^\infty\Sigma(R)\overline{v_{LOS}^2(R)}R\,dR=2\int_0^\infty\rho(r)\frac{d\Phi(r)}{dr}r^3dr$$

(3.6)
$$\int_0^\infty\Sigma\overline{v_{LOS}^4}R\,dR=2\int_0^\infty\rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3dr$$

(3.7)
$$\int_0^\infty\Sigma\overline{v_{LOS}^4}R^3dR=2\int_0^\infty\rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5dr$$

profiles

parameters

- Boltzmann equation in spherical coordinates

(3.1) $$\frac{\partial f}{\partial t}+p_r\frac{\partial f}{\partial r}+\frac{p_\theta}{r^2}\frac{\partial f}{\partial \theta}+\frac{p_\phi}{r^2\sin^2\theta}\frac{\partial f}{\partial \phi}-\left(\frac{\partial \Phi}{\partial r}-\frac{p_\theta^2}{r^3}-\frac{p_\phi^2}{r^3\sin^2\theta}\right)\frac{\partial f}{\partial p_r}-\left(\frac{\partial \Phi}{\partial \theta}-\frac{p_\phi^2\cos\theta}{r^2\sin^3\theta}\right)\frac{\partial f}{\partial p_\theta}-\frac{\partial \Phi}{\partial \phi}\frac{\partial f}{\partial p_\phi}=0$$

Integrals over momentum coordinates

Jeans equations

(3.2) $$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial \Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

(3.3) $$\frac{\partial}{\partial r}(\rho\overline{v_r^4})+\frac{2\beta'}{r}\rho\overline{v_r^4}+3\rho\frac{d\Phi}{dr}\overline{v_r^2}=0$$

(3.4) $$4\rho\overline{v_r^2v_t^2}+r\frac{\partial}{\partial r}(\rho\overline{v_r^2v_t^2})+\rho\frac{\partial \Phi}{\partial r}\overline{v_t^2}r-\rho\overline{v_t^4}=0$$

$$\beta=-(\overline{v_\theta^2}+\overline{v_\phi^2})/(2\overline{v_r^2})+1 \qquad \beta'=1-3\overline{v_r^2v_t^2}/(2\overline{v_r^4})$$

profiles

Integrals over spatial+momentum coordinates

Virial equations

(3.5) $$3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}R\,dR=2\int_0^\infty \rho(r)\frac{d\Phi(r)}{dr}r^3dr$$

(3.6) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3dr$$

(3.7) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R^3dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5dr$$

parameters

(3.2) $$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial\Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

(3.5) $$3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}R\,dR=2\int_0^\infty \rho(r)\frac{d\Phi(r)}{dr}r^3\,dr$$

(3.6) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3\,dr$$

(3.7) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R^3\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5\,dr$$

Density profiles – Jeans analysis III [6]

(3.2) $$\frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial\Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

(3.5) $$3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}R\,dR=2\int_0^\infty \rho(r)\frac{d\Phi(r)}{dr}r^3\,dr$$

(3.6) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3\,dr$$

(3.7) $$\int_0^\infty \Sigma\overline{v_{LOS}^4}R^3\,dR=2\int_0^\infty \rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5\,dr$$

Density profiles – Jeans analysis III [6]

$$(3.2)\quad \frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial\Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

- Choose parametrizable profiles for ρ_{DM}, ρ, and β

$$\left.\begin{array}{l}\rho_{DM}\rightarrow\Phi\\ \rho\\ \beta\end{array}\right\}\text{ with 3.2: }\overline{v^2}_r\rightarrow\text{projecting along LOS: }\overline{v^2}_{LOS}$$

- Infer $\overline{v^2}_{LOS}$ from observation and compare to calculation

$$(3.5)\quad 3\int_0^\infty\Sigma(R)\overline{v_{LOS}^2(R)}\,R\,dR=2\int_0^\infty\rho(r)\frac{d\Phi(r)}{dr}r^3\,dr$$

$$(3.6)\quad \int_0^\infty\Sigma\overline{v_{LOS}^4}\,R\,dR=2\int_0^\infty\rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3\,dr$$

$$(3.7)\quad \int_0^\infty\Sigma\overline{v_{LOS}^4}\,R^3\,dR=2\int_0^\infty\rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5\,dr$$

Standard Jeans analysis

Density profiles – Jeans analysis III [6]

$$(3.2) \quad \frac{\partial}{\partial r}\overline{v_r^2}\rho+\rho\frac{\partial\Phi}{\partial r}+\frac{1}{r}\rho\overline{v_r^2}2\beta=0$$

- Choose parametrizable profiles for ρ_{DM}, ρ, and β

$$\left.\begin{array}{l}\rho_{DM}\rightarrow\Phi\\ \rho\\ \beta\end{array}\right\}\text{ with 3.2: } \overline{v^2}_r\rightarrow\text{projecting along LOS: }\overline{v^2}_{LOS}$$

- Infer $\overline{v^2}_{LOS}$ from observation and compare to calculation

Standard Jeans analysis

$$(3.5) \quad 3\int_0^\infty \Sigma(R)\overline{v_{LOS}^2(R)}\,R\,dR=2\int_0^\infty\rho(r)\frac{d\Phi(r)}{dr}r^3\,dr$$

$$(3.6) \quad \int_0^\infty \Sigma\overline{v_{LOS}^4}\,R\,dR=2\int_0^\infty\rho\overline{v_r^2}\left(1-\frac{2}{5}\beta\right)\frac{d\Phi}{dr}r^3\,dr$$

$$(3.7) \quad \int_0^\infty \Sigma\overline{v_{LOS}^4}\,R^3\,dR=2\int_0^\infty\rho\overline{v_r^2}\left(1-\frac{6}{7}\beta\right)\frac{d\Phi}{dr}r^5\,dr$$

- Similarly, with the same parametrized profiles:

$$\left.\begin{array}{l}\rho_{DM}\rightarrow\Phi\\ \rho\\ \beta\end{array}\right\}\text{ rhs of above equations}$$

- Infer the lhs from observation and compare

Estimation with Virial parameters

Density profiles – Read-Steger profiles I [7]

- Based on Jeans analysis+Virial shape estimators
- Parametrization:

Stellar velocity-anisotropy

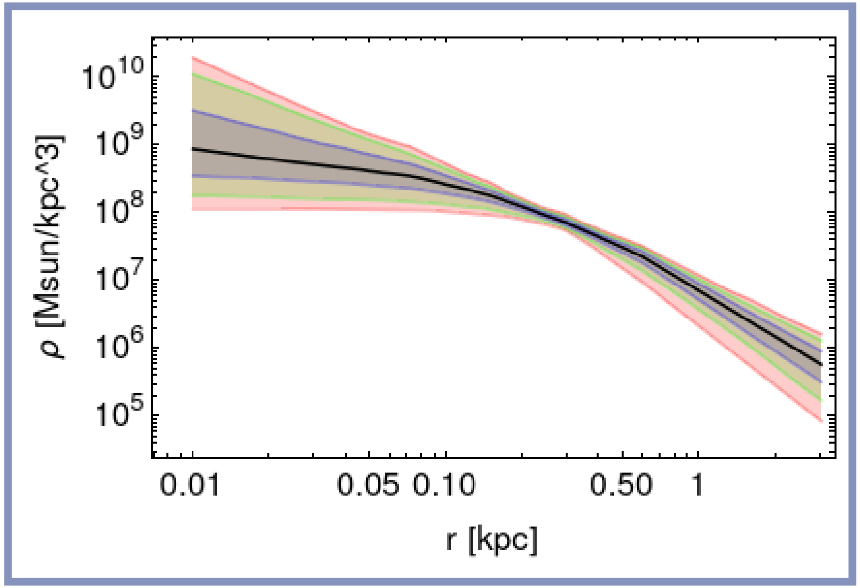
(4.1) $$\beta(r)=(\beta_\infty-\beta_0)\frac{r^2}{r_\beta^2+r^2}+\beta_0$$

N_p Plummer spheres

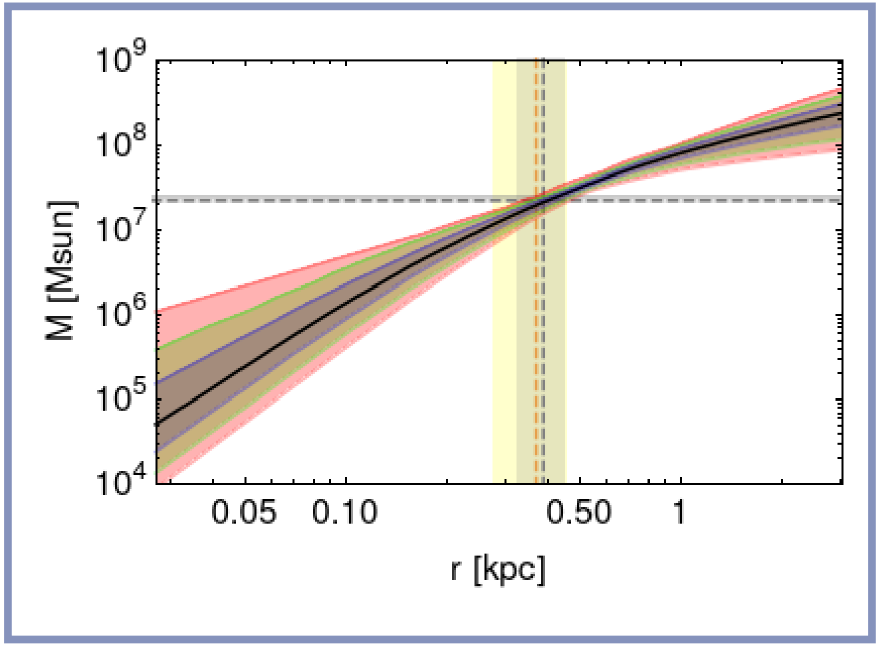
(4.2) $$\rho^*(r)=\sum_j^{N_p}\frac{3M_j}{4\pi a_j^3}\left(1+\frac{r^2}{a_j^2}\right)^{-5/2}$$

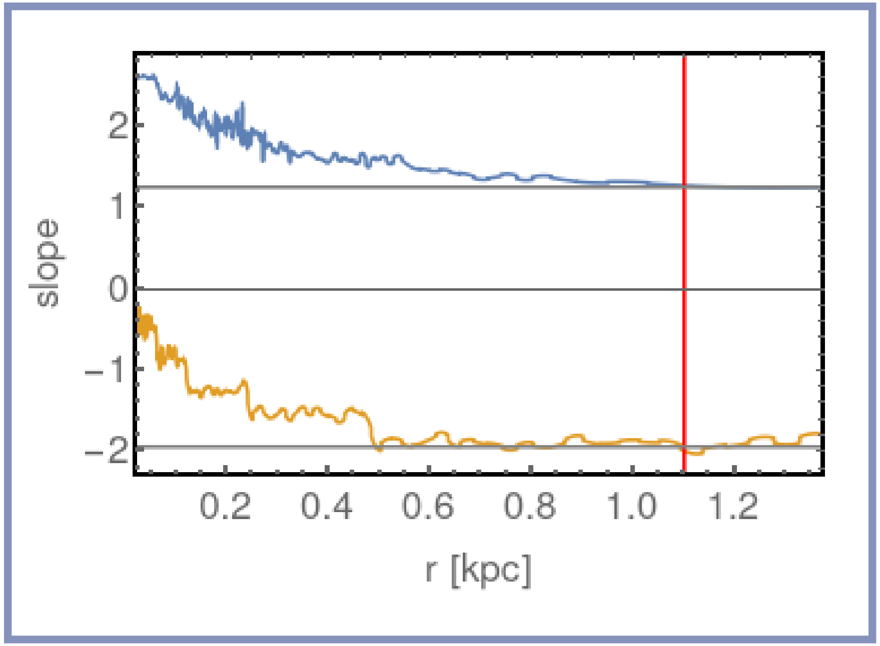
Sequence of N_{dm} power laws

(4.3) $$\rho_{DM}(r)=\begin{cases}\rho_0\left(\frac{r}{r_0}\right)^{-\gamma_0} \\ \rho_0\prod_{n=0}^{j<N_{dm}}\left(\frac{r_{n+1}}{r_n}\right)^{-\gamma_n}\left(\frac{r}{r_{j+1}}\right)^{\gamma_j+1}\end{cases}$$



Density profiles – Read-Steger profiles II





- Grey lines represent the half-light radius associated with minimal uncertainty in the mass profile.[8]
- We extend the profiles in order to get more accurate escape velocities

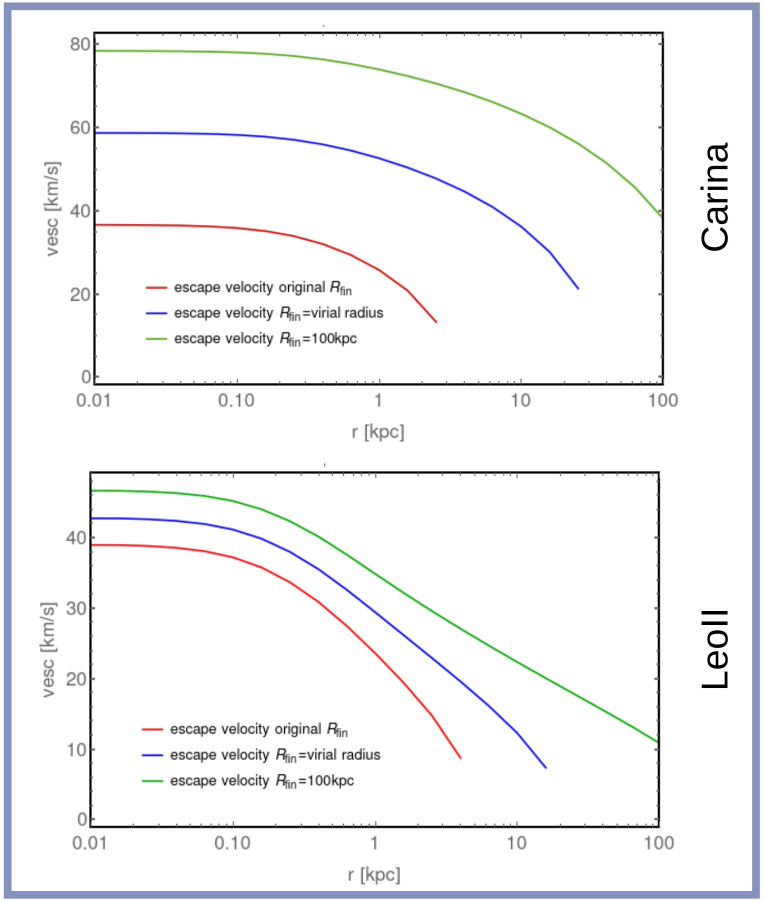
Mass Bound I

- With the Tremaine-Gunn bound from before

$$m \geq \left(\frac{\hbar^3 \, 6 \pi^2 \rho(r)}{g \, v_{max}^3(r)} \right)^{1/4} \quad (5.1)$$

- Calculate escape velocity as

$$v_{esc}^2(r) = 2G \int_r^{R_{edge}} \frac{M(r')}{r'^2} dr' + \frac{2G M(R_{edge})}{R_{edge}} \quad (5.2)$$



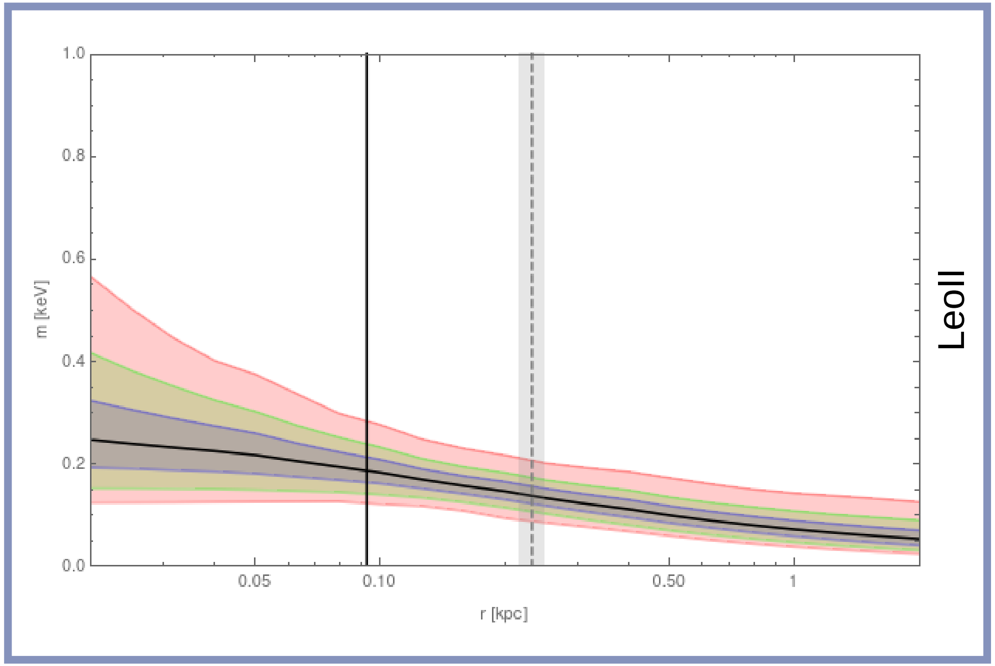
Mass Bound II

- With the Tremaine-Gunn bound from before

(5.1) $$m \geq \left(\frac{\hbar^3 6 \pi^2 \rho(r)}{g v_{max}^3(r)} \right)^{1/4}$$

- For LeoII:

(5.3) $$m \geq 187_{45}^{53} eV$$



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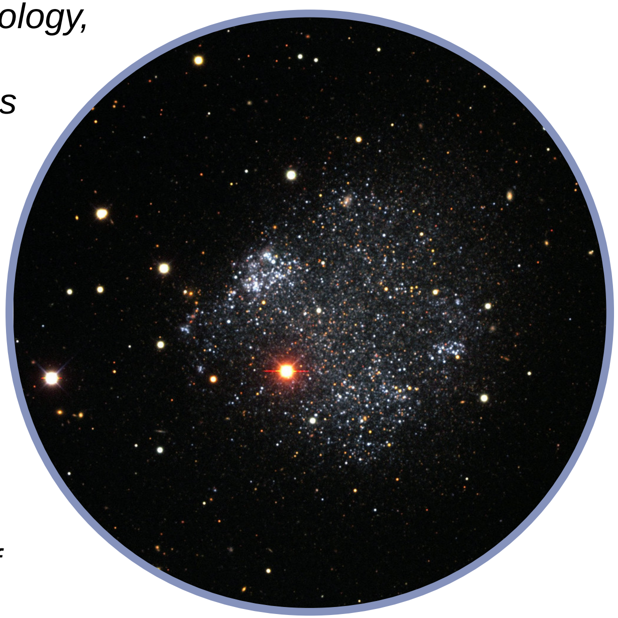
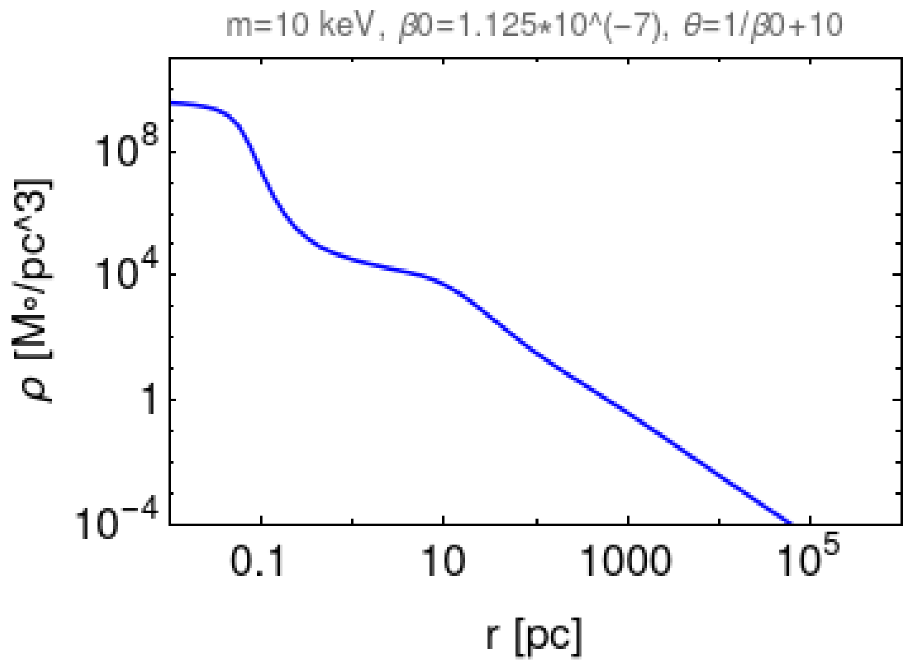
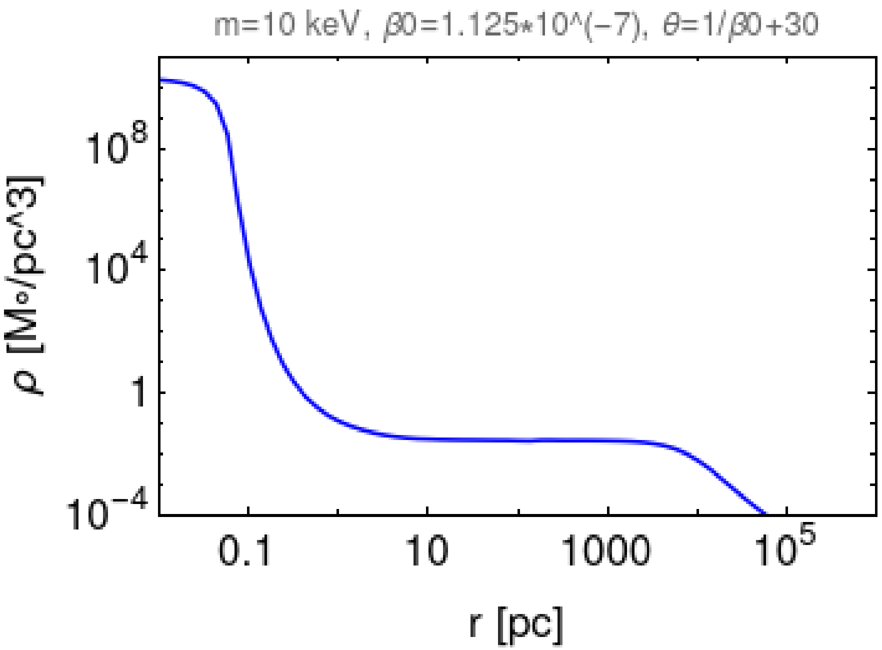



m=10 keV, β0=1.125*10^(-7), θ=1/β0+10
ρ [M∘/pc^3]
10^8
10^4
1
10^-4
0.1
10
1000
10^5
r [pc]


m=10 keV, β0=1.125*10^(-7), θ=1/β0+30
ρ [M∘/pc^3]
10^8
10^4
1
10^-4
0.1
10
1000
10^5
r [pc]

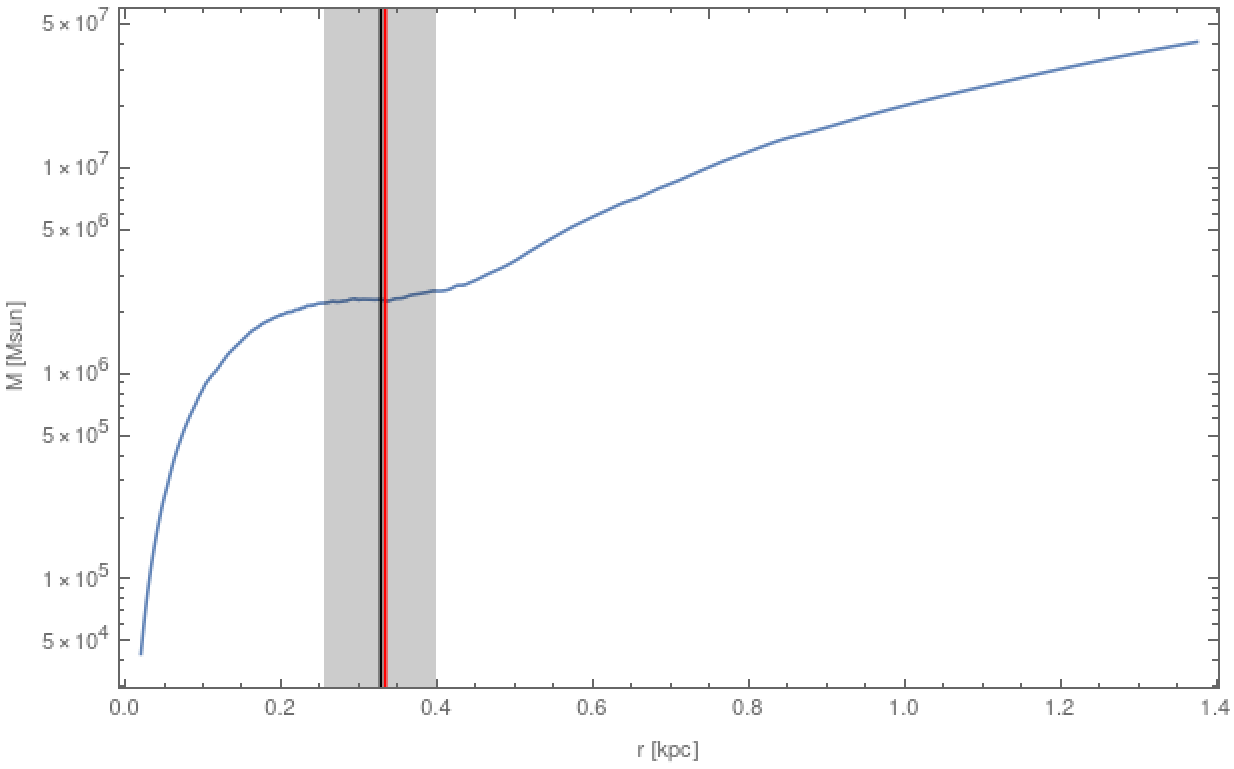
Miscellaneous II

$$\Sigma(R)\overline{v_{LOS}^2(R)}=2\int_R^\infty\left(1-\beta(r)\frac{R^2}{r^2}\right)\frac{\rho^*(r)\,\overline{v_r^2(r)}\,r}{\sqrt{r^2-R^2}}dr$$

$$\Sigma(R)=2\int_R^\infty\rho^*(r)\frac{r\,dr}{\sqrt{r^2-R^2}}$$

$$\mathscr{L}(d|p)=\prod_j^{N_b}\frac{1}{\sqrt{2\pi Var[S_{2,j}]}}\exp\left(\frac{-[S_{2,j}-\overline{v_z^2}(R_j|p)-b_j]^2}{2\,Var[S_{2,j}]}\right)$$

Miscellaneous III



Miscellaneous IV